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Game theory: Multiplayer games

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Tomáš Votruba

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Multiplayer games

Multiplayer games (or many player games or n-player games) are part of Game theory, which focuses on games where more than two players are. It is considered as games with n ≥ 2 players, and every player has d ≥ 2 strategies to choose. There is many of multiplayer games that can be easily turned into a 2-player games.

In real world the people will confront much more often with multiplayer games, than with pure 2 player games. In some way, the whole life and surviving on this planet could be stated as a multiplayer game, because every living person has to choose the way to act (strategy) and therefore affect the others.

Before we will get to the multiplayer games, let’s take a short review on basic game theory terms[[1]](#footnote-1):

* Game: Any set of circumstances that has a result dependent on the actions of two of more decision makers ("players")
* Players: A strategic decision maker within the context of the game
* Strategy: A complete plan of action a player will take given the set of circumstances that might arise within the game
* Payoff:The payout a player receives from arriving at a particular outcome. The payout can be in any quantifiable form, from dollars to utility.
* Information Set: The information available at a given point in the game. The term information set is most usually applied when the game has a sequential component.
* Equilibrium: The point in a game where both players have made their decisions and an outcome is reached.

In this chapter we will take a closer look at the most common types of multiplayer games, such as n-player prisoners dilemma, acutions etc.

# N-players prisoners dilemma

N-player prisoner’s dilemma is the game with **n-players with d = 2 strategies**: cooperate or defect. The payoff matrix is similar to the 2-players prisoner’s dilemma: The best situation for everyone is to cooperate, however if one of the players will not cooperate, he will gain significant advantage over the others. Considered that, the **dominant strategy for every player is to defect**.

Let’s explain it better on classical example “Tragedy of the commons”[[2]](#footnote-2)

## Tragedy of the commons

Picture a pasture, which herdsmen can use to feed their cattle. It is expected, that herdsmen will try to keep as many cattle as possible on the common pasture, so the pasture is full in quite a short time and adding more cattle would lead to overgrazing.

As every herdsman is rational being, he tries to maximize his profit, he asks: “What is the utility to me of adding one more animal to the herd?” The utility has one negative and one positive component:

1. The positive component is a function of the increment of one animal. Since the herdsman receives all the proceeds from the sale of the additional animal, the positive utility is nearly +1.
2. The negative component is a function of the additional overgrazing created by one more animal. Since, however, the effects of overgrazing are shared by all the herdsmen, the negative utility for any particular decision-making herdsman is significantly smaller than the positive component.

Adding together all components a rational herder concludes, that the sensible decision is to add one cattle to the herd every time. But since every herder is “locked” in conclusion of infinite adding cattle to herds, the common field will be completely destroyed and therefore there will be no pasture to feed the cattle.

What is the solution of this? According to author it seems like the changing of rules, so the “defect” is not the dominant strategy is the only solution. Garrett Hardin suggests few examples how to manage it:

* To sell the commons off as private property, or
* Allocate the right to use common property (one cow per person max, bought able right etc.)

## Today applications

Similar situations can be found today, too. Most of them could be called environmental problems. *“The commons dilemma stands as a model for a great variety of resource problems in society today, such as water, forests, fish, and non-renewable energy sources such as oil and coal.”[[3]](#footnote-3)*

There are mentioned many of potential or actual tragedies:

* Driving cars – There are many negative externalities of driving a car (pollution, carbon emission, traffic accidents etc.)
* Human population growth – Uncontrolled human population growth may lead to overpopulation (today problems can be viewed in China)
* Air – Air is highly polluted by industrial emission and other sources.
* Water – similar to air problem
* Forests – Frontier logging of old growth forests
* Radio frequencies – Unlicensed frequencies used for wireless communication are vulnerable to the overuse of high power transmitters
* Spam email – Spamming degrades the usefulness of the email system and increases costs for all users of the internet while providing a benefit only for tiny group of individuals.

# Auctions

A game-theoretic auction model is a mathematical game represented by a set of players. Generally, the players are the buyers or the sellers (auctioneers). Every player has a set of actions (strategies) available. The action set of each player is a set of bid functions or reservation prices (reserves). Each bid function maps the players value (for buyer) or cost (in case of seller) to bid a price. The payoff vector of each player under a combination of strategies is the expected utility (in case of buyer) or expected profit (in case of seller) of that player under the combination of strategies.

General desire of the buyers is to maximize their utility and therefore get the auctioned object for the lowest price possible. On the other hand the seller’s desire is to maximize his/her profit, thus selling the object for the highest price possible.

## The revenue equivalence theorem

The revenue equivalence theorem created by Riley & Samuelson in 1981 states that any allocation mechanism/auction will lead to the same expected revenue for the seller if the mechanism/auction meets following:

1. The bidder with the highest valuation always wins
2. The bidder with the lowest possible valuation expects zero surplus
3. All bidders are risk neutral
4. All bidders are drawn from a strictly increasing atomless distribution

There are various types of auctions, depending on the rules of placing bids. The most common 4 types are:

## English auction

English auction, also known as open ascending price auction is one of the most common types of the auction. The participants makes increasingly higher bids. The bids are public, so everyone participating, knows the current highest bid. It is assumed, that that participant, stops increasing the bids, when the price reaches maximum price, he/she is willing to pay for the object.

Sometimes the starting prices may be set by seller or the object is sold, only if the price reaches reserve price set by seller.

The buyers dominant strategy for this type of auction is to remain in the auction until the asking price is equal to his value. If we consider the case of two buyers, each with a value, that is independent draw from a distribution with support [0, 1], cumulative distribution F(v) and probability density function f(v). If buyers bid according to their dominant strategy, then a buyer with value v wins if opponents value x is lower. [[4]](#footnote-4)

Therefore the win probability is:

and the expected payment is:

Then the expected payment conditional upon winning is:

## Dutch auction

The starting price is set by the auctioneer at level sufficiently high to deter all bidders. Then it is progressively lowered until one of the bidders calls. The one, who called is winner of the auction.

## First-price sealed bid auction

In this type of auction, the bidders place their bid in a sealed envelope and hand them to the auctioneer at one time. Then the envelopes are opened and the bidder with the highest bid wins the auction.

## Vickrey auction

Vickrey auction is also called a second-price bid auction. The bidders place their bid in a sealed envelope and then handed to the auctioneer. The auction wins the bidder with highest bid, but he has to pay the amount of the second highest bid.

There are also other special types of auctions:

## All pay auction

In this auction the bidders place their bids in the sealed envelope and hand it to the auctioneer. The envelopes are opened and the auction wins the one with the highest bid, but every bidder has to pay the amount he or she bided.

## Amsterdam auction

Starts as English auction, until there are only two bidders. Then the auction switches to First-price sealed auction, for the remaining two bidders.

# One dollar auction: Escalation of commitment

The dollar auction is game presented by economist Martin Shubik. The auction is for one dollar bill. The bid is called globally and the minimum increment price is 10 cents. The winner of the auction is the highest bidder, who pay the amount he bids. In addition the second highest bidder must also pay his/her bid, but gets nothing in return.

The trick in this type of game is, that it is more efficient for you to bid every time, if you are the second highest bidder. Let’s say we are being outbid on 70 cents and the current highest bid is 80 cents. We get in position of the second highest bidder and we have two options:

1. Fold the game and pay the 70 cents or
2. Bid 90. That would lead to 90 cents profit if we would win.

The obvious step is to bid 90 cents. But that would make 80 cents-bidder the second highest bidder and he/she has two options too:

1. Fold the game and pay 80 cents or
2. Bid $1 and therefore have no profit and no loss.

The obvious action is to bid and that would throw us in the same position as we were before:

1. Fold the game and pay 90 cents or
2. Bid $1.10 and pay only 10 cents

When the auction jumps over $1 there is no chance to profit on the auction, but players are still forced to bid more and more, just to have lesser loss than the opponent. The game ends, only when one of the players makes “irrational” decision and folds the game.

According to the article in Man vs Logic[[5]](#footnote-5), this game was often played at Stanford university in the economics classes and it was not uncommon to see the game end anywhere between five and ten dollars.

Although the Escalation of commitment is mostly the thought experiment, there are several examples in the real life where are the same or very similar rules:

* *“After a heated and aggressive bidding war, Robert Campeau ended up buying Bloomingdale's for an estimated 600 million dollars more than it was worth. The Wall Street Journal noted that* "we're not dealing in price anymore but egos." *Campeau was forced to declare bankruptcy soon afterwards.”[[6]](#footnote-6)*
* *“Supporters of the Iraq War have used the casualties of the conflict in Iraq since 2003 to justify years of further military commitment. This rationale was also used during the sixteen-year Vietnam War, another military example of the logical fallacy.”*2

# Multiplayer cooperative games

The cooperative game is a game where two groups of players (coalitions) may enforce cooperative behavior. The game is rather a competition between coalitions of players rather than between players themselves.

## Coalitions

Coalition is a subset of the set of players. Coalition forms in order to coordinate strategies and to agree on how the total payoff will be divided among the members.

Let *P* be the set of the players and there are N players in the system, the N is also called a *grand coalition*. A coalition is denoted by an uppercase script letters: *S, T, U* etc. Formed coalition S ⊆ P and the counter coalition *SC*is:

In general, if there are N players in the game, there are 2N possible coalitions.

Example of 3-player game:

There can be 23 = 8 possible coalitions:

* The grand coalition itself:
* 3 one-player coalitions:
* 3 two-player coalitions:
* Empty coalition: ∅

## Characteristic function

The simple way to look at cooperative games is to view a competition between two “players”: Coalition S and the counter coalition SC.

Let’s consider an N-player game, where *P* = {P1... PN} and Xi is the strategy set for player Pi. The game also has no empty coalition. Therefore we can create a bi-matrix with rows and columns that correspond to the pure joint strategies of players in *S* (or *SC*). The matrix has entries as pairs of numbers. Where the first number is the sum of payoffs of the coalition S and the second one is the sum of payoffs of the counter coalition *SC*.

The maximum value for the coalition *S* in bi-matrix is called the characteristic function of S and is denoted as v(*S*). [[7]](#footnote-7)

## Superaditivity theorem

The theorem of Superaditivity could be vaguely explained as: “There is strength in union.” It states, that the value of union is no less than the sum of coalitions separate values:

Let *S* and *T* be disjoint coalitions:

## Imputation

Suppose a coalition formed in an N-person game. Every player in this game wants to know, how much he/she gains, if he forms a coalition. The amount of profit going to the players forms an N-tuple x of numbers. That N-tuple vector x must satisfy two conditions:

* Individual rationality: For all players
* Collective rationality: We have

If the N-tuple satisfies the conditions, it is called then an imputation.

## The Core

If there is an imputation, which is dominated through some coalition, it will never become permanently established. There will always be tendency for this coalition to break up and be replaced by one, which gives its members a larger share.

The imputations that are not dominated by any other imputations through any coalition are called the core.

*“If an imputation x is in the core, there is no group of players which has a reason to form a coalition and replace x. Therefore, the core is the “solution concept” of N−person cooperative games. As we will soon seen, this solution concept is ok as long as the core is not empty.”7*

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